Topological Analysis of Foams and Tetrahedral Structures**

By *Gad Frenkel,* Raphael Blumenfeld, Peter R. King* and *Martin J. Blunt*

The analysis of morphology of porous materials, such as granular aggregates and foams, has advanced greatly in the past decade due to increasing computational abilities and micro-tomography tools.[1,2] The study of low porosity materials, such as soil, is significant to the oil industry[3] due to interest in permeability, heat conduction and improved recovery.[4] Many algorithms have been suggested with limited success for extracting pore networks that describe the porous structure properly.[1,2,5,6] The mechanical properties of the porous solid are also of great interest in the granular materials community[7] and in the porous metal community.[8] The combination of high porosity, high surface area, solid-like mechanical properties, and low mass in materials such as metallic foams are important to many applications, e.g., heat exchange[9,10] and joint replacements and additional medical applications.[8,11] Several approaches have been used to study the morphological structure in attempts to provide meaningful descriptors of cellular structure. Foams have been mostly studied using either known periodic structures[3,12,13] or by simulated structure, e.g., by the software surface evolver.[14,15] From these, simple properties were extracted from images. In the past few years, methods of extraction of morphological properties and skeleton networks from three-dimensional (3D) tomography have been developed.[1,2,10,16,17] Among the advantages of such approaches are the direct applicability to real systems and the ability to monitor the properties of particular samples. For example, Vicente and coworkers[18] have studied the 3D structure of metallic foams, using the watershed algorithm, to obtain two interleaved skeletons for the cells and the solid matrix. Another approach is to use the 3D images to obtain statistical descriptors of the system, using integral-geometry morphological image analysis, that can characterize patterns in terms of Minkowski functionals.[19] The Minkowski functionals produce several scalar structural descriptors that are complimentary to two-point correlation functions. While, these descriptors are simple global properties it is not trivial to connect these descriptors to the local structure in terms of pore-scale networks. The Minkowski functionals are part of the data that describes statistically the structure of the system. Still, the novelty and complex structure of these materials require further investigations of the morphology topology and different and more involved statistical properties. In particular, most existing network descriptions fail to quantify the connectivity, a key property in porous materials. Therefore, fundamental theories predicting the properties of porous structures are high in demand. The aim of this paper is; (i) to introduce a promising approach – the Blumenfeld–Edwards entropic formalism,[20,21] (ii) to present a method for identification of the cellular structure, (iii) to represent the cellular data in a way that is compatible with the Blumenfeld–Edwards formalism, and (iv) to analyze simple properties of cellular structures.

Foams and granular media have common properties. Both have disordered structure, for which the void space is as important as the material, and whose characterization is a key to the understanding and prediction of physical properties, such as permeability and thermal or electrical conductivity. These similarities suggest that the structure and properties of these systems may be described and analyzed within a unified formalism. Such a formalism has been proposed by Blumenfeld and Edwards,[20,21] following earlier suggestions for applying an entropic formalism to granular systems.[22] The formalism is analogous to statistical mechanics in conventional thermodynamic systems, where the energy of the system is replaced by its volume and the temperature is replaced by a new quantity, called compactivity.[22] A key to its application is the description of the configurations of the system by a well defined set of parameters, called degrees of freedom.

In this paper, we illustrate the application of the unified method on systems that can be represented as a set of connected tetrahedra: quadrivalent foams (having four plateau borders per vertex) and 3D Voronoi tessellations. The method consists of first transforming the foam skeletal network into a connected assembly of pseudo-grains. Next, an automated method is described to identify the cellular structure. The cellular structure is then used to extract topological data, such as the number of skeletal throats per cell. Finally, we demonstrate how to use the Blumenfeld–Edwards formalism to compute expectation values of structural properties of model three-dimensional structures.

[*] Dr. G. Frenkel, Dr. R. Blumenfeld, Prof. P. R. King,
Prof. M. J. Blunt
Earth Science and Engineering, Imperial College
London SW7 2AZ, UK
E-mail: g.frenkel@imperial.ac.uk; gadyfr@hotmail.com
Dr. R. Blumenfeld
Biological and Soft Systems, Cavendish Laboratory
Cambridge CB3 0HE, UK

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Structural Characterization

At the heart of the characterization method is a partition of the void space into cells, inter-connected via skeletal throats sometimes referred to as "windows." The foam structure is transformed into a framework of polyhedra as shown in Figure 1(c–e). The transformation preserves the correct connectivity of both the lamella of the foam and of the void space. There are various methods to identify the plateau borders and vertices of the foam.[4,17,23] One method successfully applied for foam structure used the watershed algorithm.[17] Other methods include methods that are based on the maximal ball algorithm[5,6] and methods that are based on the medial axis algorithm.[23] These methods have been applied successfully to micro-tomography images to extract the pore network of low porosity materials.[2,5] The maximal ball method assumes that pores are locally wider than the throats that link them to neighboring pores. The algorithm, and its successors, are designed for low porosity and thus are ideal for the identification of the vertices and plateau borders. For foam the algorithm acts upon the lamella of the foam in which the pores, found by the algorithm, represent the vertices and the throats represent the plateau borders. While, this method is applicable to general structures, in what follows we concentrate on $Z_V = 3$ in two-dimensional (2D) and $Z_V = 4$ foam in 3D. We also assume in the following that the vertices and plateau borders have been properly identified.

Consider a foam, whose every vertex is connected to exactly four other vertices by plateau borders, $Z_V = 4$. As a first step, every vertex is "dressed" by a tetrahedron whose corners are the midpoints of the four plateau borders connecting it to its neighbors (Fig. 1d). The tetrahedra touch their neighbors at the corners. We can regard the tetrahedra as pseudo-grains and their corners as the inter-granular contact points. Topologically, the foam and the tetrahedra structure are homeomorphic, i.e., the connectivity of the two structures and of their void spaces are identical. A similar transformation can be carried out for 2D trivalent ($Z_V = 3$) foams, where vertices are dressed by triangles by connecting the midpoints of the ligaments around them (Fig. 1a, b).

Cells are simply-connected polyhedra (Fig. 2), homeomorphic to a sphere. The surface of a cell consists of two types of facets: the triangular facets of the tetrahedra surrounding the cell (opaque triangles in Fig. 2), which in the following we refer to as faces, and the skewed polygons that the triangles enclose (transparent in Fig. 2). The triangular faces form a close network on the surface – every face is connected to exactly three other faces on the same surface. By construction, every facet can be associated with exactly one cell and one tetrahedron. Thus, there are exactly four cells around each of the tetrahedra, corresponding to the four bubbles that meet at the vertex of the foam. The cell, defined by the facets, is open and connected to its neighboring cells via the aforementioned skewed polygons, which we call skeletal throats. A skeletal throat is characterized by the tetrahedra edges that make its perimeter.

The partition of the foam into cells and skeletal throats is unique, namely, following the above rules every void point will belong to one and only one well-defined cell. Failing to identify a face would result in an open set of faces around a particular cell. In practice, the characterization method matches the cell to the original bubbles. The uniqueness of the partition, correspondence to the physical cells, and the lack of ambiguity is a major advantage of this characterization method.

Since a cell is a polyhedron homeomorphic to the sphere, then its properties are related by Euler relation,[24]
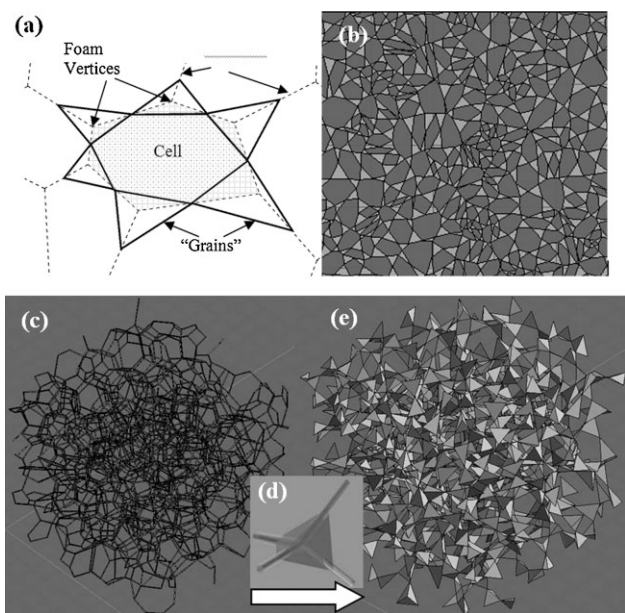
$$n_c - n_e + n_f = 2, \tag{1}$$

Fig. 1. Transformation of foams to pseudo-granular structures. a) Six vertices of a 2D foam, forming a cell, are "dressed" by triangles (full lines) by connecting the midpoints of the ligaments around them (dashed lines). b) Part of a transformed 2D foam. c) The plateau borders and vertices of a 3D foam. d) Transformation of the vertices into tetrahedra by connection of the midpoints of the plateau borders. e) The corresponding network of tetrahedra.
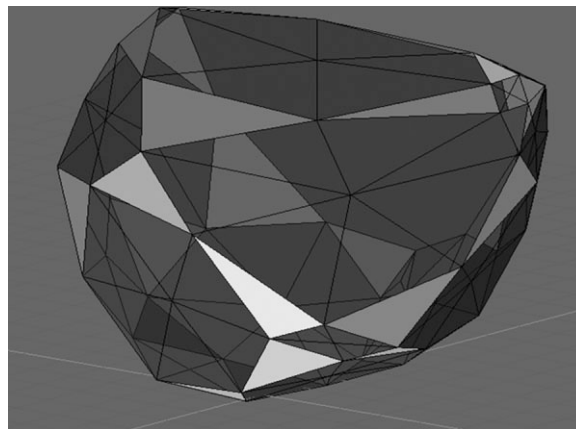

Fig. 2. A typical cell. The surface consists of a closed set of triangular faces of the tetrahedra that surround the cell (opaque triangles) and skewed polygons (transparent). The latter are the skeletal throats between neighboring cells.

where n_c is the number of corners, n_e the number of edges, and n_f the number of facets. Within our construction, a cell having Z_f vertices and Z_t skeletal throats surrounding it would have $3Z_f$ edges and a total of $Z_t + Z_f$ facets. Since each corner is shared between two edges then the number of corners is half the number of edges, $3Z_f/2$. Substituting in Euler relation (1), gives

$$Z_t = 2 + \frac{Z_f}{2} \qquad (2)$$

It is interesting to note that Z_f, the number of triangular faces surrounding a cell, must be even, which also follows from the fact that the number of corners is integer. This also means that the number of vertices around a bubble of the original foam must be even.

Fig. 4. Illustration in 2D of cell identification by expansion of non-concave deformable bubbles: a bubble (closed dark loop) fills the cell and protrudes out of it only minimally due to its non-concavity.

Identification of Cells and Skeletal Throats

As mentioned, skeletal throats are the skewed polygons between cells. Once the set of faces is known, the skeletal throats are identified as follows. Place normal unit vectors on every face, directed outwards from its tetrahedron and draw vectors around the face in the clockwise direction when seen from inside the tetrahedron (Fig. 3). The boundary of a skeletal throat is identified by following loops of the same vectors in the anticlockwise direction.

We now need to identify the set of faces that surround a given cell. To this end, we use a different set of algorithms from the one used for the lamella. This is because the maximal ball algorithm is designed for low porosity while the porosity

in foams is very high and because the skeletal throats are no longer tube-like structures, which makes that algorithm less effective. Instead, we use an algorithm that involves inflation of a deformable bubble from the center of a facet into the cell, while keeping the bubble's curvature positive everywhere at all times. This ensures that the inflation process stops soon after the bubble starts exiting the cell through the throat (Fig. 4). The inflation can be done, e.g., by imposing that the bubble may not penetrate any grain and may have no negative curvature anywhere, or by treating it as a physically growing balloon, following Hamiltonian dynamics with a curvature energy term that disallows grain penetration and favors a positive surface curvature everywhere. An advantage of the latter method is the ability to implement different energy terms and to mimic real physical processes, such as mercury injection. This method works best with a 3D voxel representation of the pore vicinity.

A more efficient cell-identification method uses a plane cutting process and produces a convex polyhedron as follows. Choosing a face, a small spherical bubble is attached to its center and is inflated away from it by moving the bubble center along the normal to the face surface. The expansion stops when the bubble cannot expand without penetrating a grain. Next, faces of vertex tetrahedra are ordered by their distance from the center. Following this order, each face added to the list defines a plane and all tetrahedra on the other half-space of this plane are deemed as not belonging to the bubble and to the list. The process ends when all the faces have been visited. The bubble comprises the intersection of all allowed half-spaces of these cutting planes. The process produces non-concave polyhedral bubbles and these form the basis for a search for the facets that belong to the cell.

Both methods produce non-concave bubbles. However, cell surfaces may have concave regions. In such cases, the set of faces surrounding the bubble may not necessarily close. Thus, the set of faces is next checked for completeness as follows. For each face, its neighbor faces are identified: these are faces of different grains that share a corner with the current face. Faces having fewer than three neighbors are disregarded recursively until either a closed, or an empty, set are left. If the set is incomplete, neighboring faces to those in the list are added and completeness is checked again. The process is
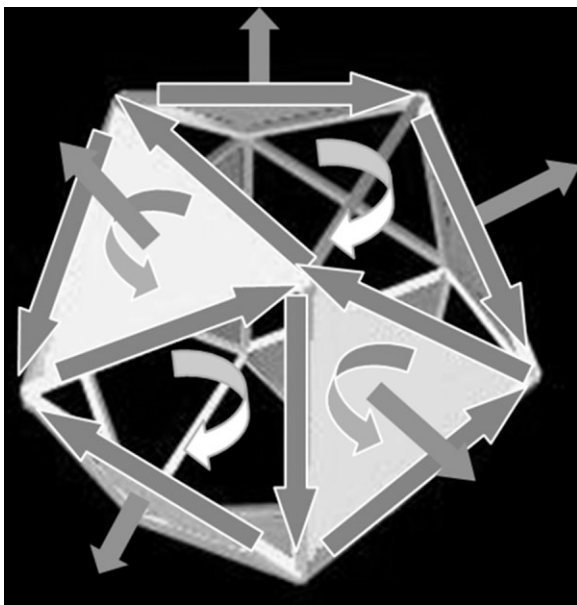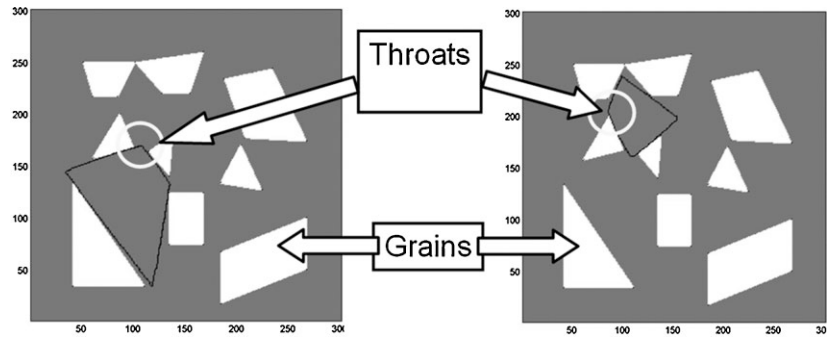

Fig. 3. Identification of skeletal throats: normal unit vectors (blue) are placed on each face, facing outwards from the tetrahedron. Vectors ordered on the boundary of faces in the positive direction of rotation around the normal vectors. The window edge is extracted by following the same vectors in the negative direction of rotation.

repeated until completeness is achieved. Then the set defines a cell and the tetrahedra whose faces belong to the complete set are identified as surrounding the cell. If more than one cell emerges at the end of the process, then some tetrahedra have more than one face in the list. In this case an additional search method is used to isolate the correct cell. A face is deleted from the set randomly and the above method is used to check for completeness again. If the resulting subset still has multiple faces that belong to the same tetrahedron or is not closed, another face is deleted and so on. Faces belonging to one cell cannot be a part of another and are excluded in following searches to improve efficiency.

To illustrate the method, we have constructed two samples: a 3D Voronoi tessellation, made by seeding points with a Poisson distribution[25] and a mono-disperse foam produced by the surface evolver and designed to have a narrow distribution of bubble volume.[15] For both samples we have constructed the tetrahedra and identified cells, using the above algorithm, as shown in Figure 5 for the Voronoi system. In addition to the tetrahedra (5A) and cell structures (5B), we show in 5C the connectivity network of cells. The latter are represented as spheres, whose sizes are proportional to the cell volumes. The tubes connecting spheres represent inter-cellular throats.

From this representation, we can extract a range of structural properties, as demonstrated in Figure 6 for a Poisson–Voronoi tessellation and a mono-disperse foam. The analysis of these properties shows that there are distinct differences between the two systems. We have studied distributions of both topological properties: the number of edges per throat, the number of throats per cell and the number of facets per cell, and of geometric properties: throat area and cell volume. We find that the topological properties of the mono-disperse foam are consistently more narrowly distributed. We also find that the distributions of the geometric properties differ significantly between the two structures. This suggests that these distributions can be used as sensitive fingerprints of foam and cellular structures. It would be very interesting to compare our results with measurements of real foam samples. Such comparisons, however, will be the subject of future work.

The topological properties of the tetrahedral structure, such as the number of throats per cell and the connectivity, can be used to extract additional geometrical properties of the original foam, such as throat cross-sectional areas of foam and contact area between a cell and the plateau borders that bound it. For example, knowing the tetrahedra/vertices that bound a window in a wet foam, the throat area can be extracted by identifying the set of plateau borders surrounding the throat using the maximal ball algorithm: the vertices of the foam are set as the local maximal balls and the plateau borders are found by the maximal ball algorithm to be the throats of the algorithm that connect them (Fig. 7). The throat area can be extracted by the projection of the plateau borders onto the plain that is normal to the vector connecting the center of the two cells. In similar manners properties such as cell volume and the cell's
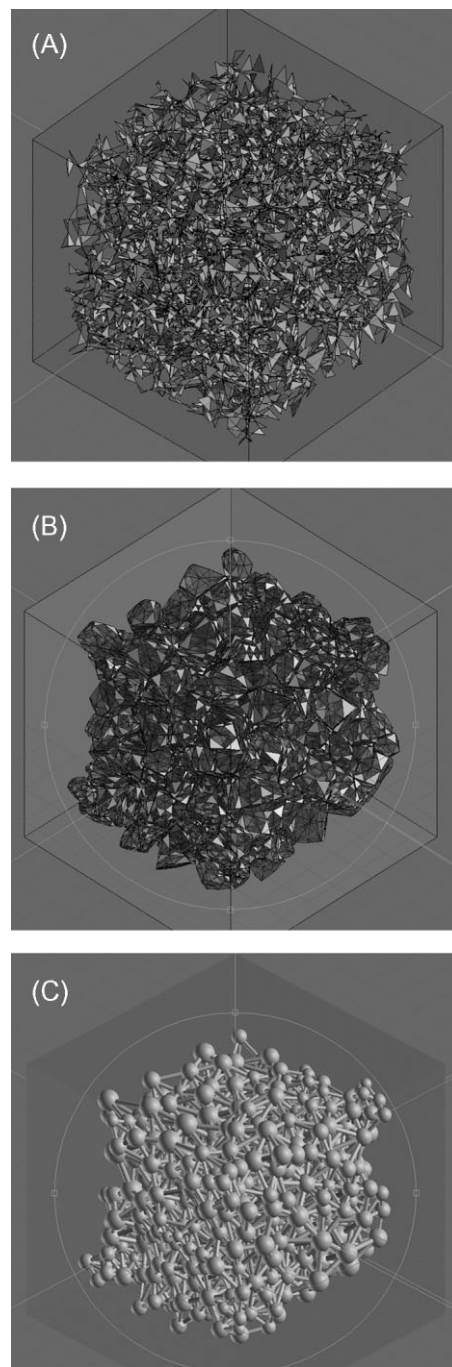

Fig. 5. Extraction of the cellular structure and the connectivity of a random Voronoi foam. A) The tetrahedra structure; B) the partition into cells (a single cell of which is shown in Fig. 2); C) the extracted network, where cells are represented as spheres, whose sizes are proportional to the cell volumes, and tubes represent the inter-cellular skeletal throats.

surface area can be extracted. We will not pursue this extraction any further in this paper as our aim here is to present the topological characterization of the structure.

Edwards' Compactivity

Finally, this method can be used to underpin the entropic formalism based on Edwards' compactivity concept.[22] It has
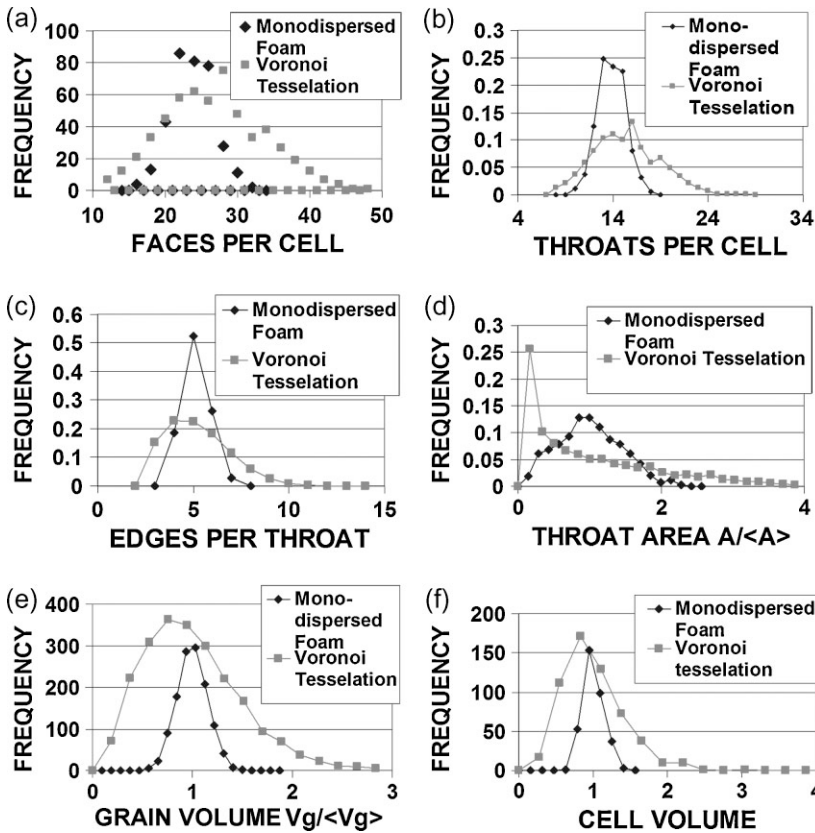
Fig. 6. Extracting data from the cellular structure of Voronoi tessellation and mono-dispersed foam. a)–c) Topological properties a) the number of faces per cell b) the number of throats per cell c) the number of edges per throat. d)–f) Morphological properties: d) the throat area normalized by the mean of the throat area e) the grain volume, normalized by the mean grain volume f) the cell volume frequency.
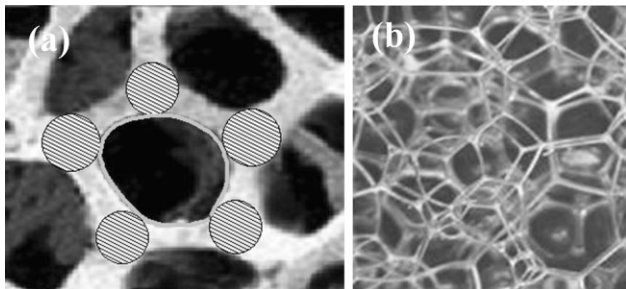

Fig. 7. a) Use of the topological information obtained and the maximal ball method to obtain the throat of metallic foam (illustration). Vertices (maximal balls) and plateau borders (throats of the algorithm) corresponding to the tetrahedra that bound a skeletal throat are identified using a maximal ball method. The structure surrounding the throat is projected to the plane, normal to the vector connecting the centers of cells that share the throat. The throat cross-section area is obtained from the projected image. b) Dry foam, for dry foam the throat area can be estimated the same way or simply by creating the skewed polygon that connects the vertices of the throat found by the topological analysis and by projecting the skewed polygon as above.

been proposed that disordered configurations of granular assemblies can be described by a statistical-mechanical-based entropic analysis, much like conventional thermodynamic systems. This approach has been later adapted to foams and cellular structures.[20,21] In this section, we give a brief review of the salient points that are relevant to the analysis to be presented below.

The formalism is based on regarding the volume of the assembly, granular systems or foams, in the same way that energy is in thermodynamic systems, and replacing the temperature by an analogous quantity—compactivity.[20–22] In this approach the structure is presumed to have a very large number of possible "states," where each state represents a possible configuration of the system components. For consolidated granular assemblies, a state is defined by the position of grains, orientation, and shape. For foams, a state is defined by the cellular structure, the connectivity of cells and so on. The subset of independent parameters that define all of the possible states is called the set of degrees of freedom. Each state has a probability of occurrence that depends on the system's volume V and states with identical volume are assumed to have equal probabilities of occurrence. The probability is given by a Boltzmann-like factor, $\exp(-V/X)$, where X is the analog of the temperature, named the compactivity.[22] Due to the astronomically large number of states in macroscopic systems, the phase space of the system's states can be regarded continuous and the states have an occurrence probability. This probability, $\Theta(\{q\})$, depends on the statistical distribution of the degrees of freedom, $\{q\}$, and it is the analog of the density of states in thermal systems. The density of states may be non-analytic because it must vanish for non-physical states.

The partition function is a non-normalized integral over the probabilities of all possible states of the system, which are expressed by the set of parameters $\{q\}$,

$$Z = \int e^{-W(\{q\})/X} \Theta(\{q\}) \Pi dq \qquad (3)$$

Here, W is a function that gives the volume of the entire system in terms of the degrees of freedom $\{q\}$. The entropy, $S(V)$, is defined as the logarithm of the number of states at a given volume $W = V$. The compactivity is defined in terms of the entropy, $X = \partial W / \partial S$, in analogy to temperature in thermal systems, $T = \partial H / \partial S$, where H is the Hamiltonian. This approach makes it possible to calculate structural properties as expectation values. For example, in consolidated materials made of grains, the mean volume associated with one grain is

$$\langle V_g \rangle = \frac{1}{Z} \int V_g(\{q\}) e^{-W(\{q\})/X} \Theta(\{q\}) \Pi dq, \qquad (4)$$

and the variance of this volume is given by

$$\begin{aligned} \mathrm{var}(V_g) &= \left\langle V_g^2 \right\rangle - \left\langle V_g \right\rangle^2 \\ &= \frac{1}{Z} \int V_g^2(\{q\}) e^{-W(\{q\})/X} \Theta(\{q\}) \Pi dq - \left\langle V_g \right\rangle^2 \end{aligned} \qquad (5)$$

The usefulness of this formalism to granular materials has been validated experimentally,[26,27] but the suggestion to apply it to cellular systems and foams[21] has not been tested. If this approach is indeed useful for the latter materials then it makes possible useful predictions of their structural properties. To check the formalism and use the partition function in Equation (4 and 5), one must identify first the degrees of freedom. The degrees of freedom are only a subset of all the possible variables that can describe the structure – the independent variables.

It has been proposed recently[20,21] that a useful set of such variables is given by small volume elements called quadrons.[21] These volume elements are quadrilaterals in two dimensions and octahedra in three. In granular systems, the identification of quadrons is based on the construction of the network of contacts between grains. In foams and cellular systems, it is based on a construction of equivalent frameworks of tetrahedra between vertices of the skeleton of the structure. Here, we use this idea to build a fully automatic method to obtain the cellular structure; to obtain the structural characteristics of the foam; and to identify the quadrons. In principle, the coordination number of a foam vertex, defined as the number of vertices it connects to, Z_V, can be arbitrary. However, for most foams and cellular structures $Z_V = 4$ and, for simplicity, we focus here only on such systems. Such systems, which we call tetrahedral systems, can also occur on the molecular level, e.g., in silicates, and in dense colloidal aggregates.

To apply this formalism to our tetrahedral structures, it is convenient to tessellate the space that the structures occupy with shapes of a particular topology. This has been done in[21] – the 3D volume of the structure is tessellated by volume elements, called quadrons, all of which are octahedra. The quadrons are constructed as follows (Fig. 8). Choose a face of a particular tetrahedron v, residing around cell c. The three edges of the face border on three polygonal throats, one of which let us call p. Connect the end points of the edge that borders p both to the centroid of the face center and to the centroid of the throat p. The edge ends and the two centroids form a skewed quadrilateral. Now connect the four corners of this quadrilateral both to the centroid of cell c and to the centroid of tetrahedron v. The result is an octahedron, which can be indexed by cvp. This is the volume element that we call quadron. It is straightforward to convince oneself that by carrying out this tessellation around every edge of every face, the tessellation covers the entire volume of the structure. Typically, quadrons are asymmetric and non-convex.

In foams there are three quadrons per face and hence twelve quadrons per tetrahedron. It has been shown that the number of degrees of freedom required to describe the structure uniquely, N_{dof}, can be bounded, $27N_V/5 < N_{dof} < 11N_V/2$, where N_V is the number of foam vertices (tetrahedra).[28] Thus, the number of quadrons ($12N_V$) is much larger
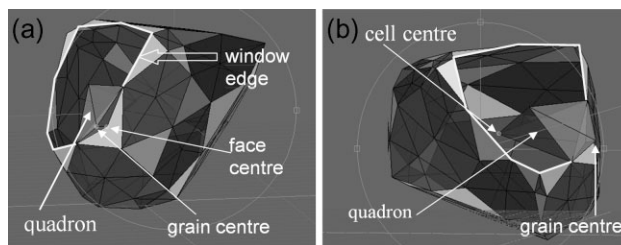

Fig. 8. Construction of quadron in 3D: an edge of a face is chosen. Its ends are connected to the centroids of both the face and a skeletal throat (the highlighted line represents the edge bordering with the skeletal throat), making a skewed quadrilateral. The corners of the quadrilateral are connected to the centroids of the tetrahedron and of the cell. The result is an octahedron – the quadron.

than N_{dof}. It is convenient to use quadrons as the degrees of freedom, but a subset of independent quadrons must be chosen. The selected quadrons may ''interact'' with one another, i.e., be correlated, which would lead to a complex density of states $\Theta(\{q\})$. The derivation of the density of states in any model is an interesting and challenging problem that has not been addressed yet in the literature. To make progress, we use here the simplest possible model: we assume that the independent quadrons are uncorrelated. This is the analog of ideal gas approximation in standard thermodynamics. Using this approximation, the partition function (3) is

$$Z = \int e^{-\alpha \sum_{q=1}^{N_{dof}} V_q / X} \Theta(\{q\}) \prod_{q=1}^{N_{dof}} dV_q$$

$$= \left(\int e^{-\alpha V_q / X} P(V_q) dV_q \right)^{N_{dof}}, \tag{6}$$

where $\alpha = 12N_v/N_{dof}$ is a factor that takes into consideration that we sum in the volume function only over a subset of the quadrons. Figure 9 shows the probability density function of quadron volumes for both the Voronoi and the mono-disperse foams. For example, the γ distribution fits well the distribution of quadroon volume for the mono-dispersed foam. Using a gamma distribution for the mono-dispersed foam,

$$P(V_q) = \frac{b^a}{\Gamma(a)} V_q^{a-1} \exp(-bV_q), \tag{7}$$

in the canonical partition function (6), and assuming the ideal quadron gas approximation, we can obtain explicitly the nth
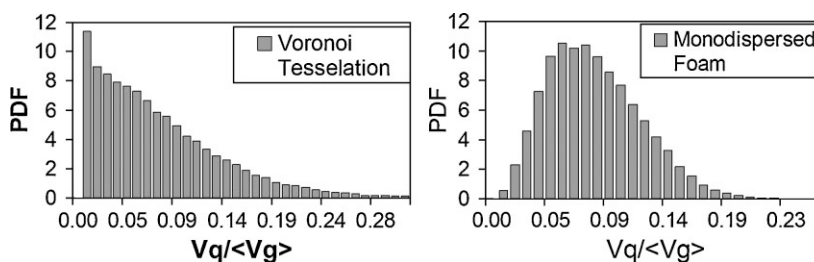

Fig. 9. The probability density function of 3D quadron volume for: Voronoi cells (left) and a mono-disperse foam (right).

moment of the distribution of the volume per quadron.

$$
\begin{aligned}
\langle V_q^n \rangle &= \frac{\int e^{-\alpha \sum_q V_q \big/ X} \sum_q V_q^n \prod_q P(V_q) dV_q}{\int e^{-\alpha \sum_q V_q \big/ X} \prod_q P(V_q) dV_q} \\
&= \frac{\Gamma(n+a)}{\Gamma(a)} (b + \alpha/X)^{-n}
\end{aligned}
\tag{8}
$$

It should be noted that, due to the normalization within this approximation, Edwards compactivity X is scaled down by a factor of α.

For the Voronoi foam, properties can be computed either numerically or by fitting the probability density function with a known function and carrying out an analytic calculation.

Conclusions

In this paper, we presented a characterization method of foams and tetrahedral cellular structures. The method comprises several steps. First, vertices (nodes) of the foam are transformed into tetrahedral pseudo-grains. This yields a framework of tetrahedra that enclose polyhedral cells. The cells form an interconnected void structure, with neighboring cells connected by skewed non-planar polygons that form skeletal throats. The topological definition of cells leads to a unique description and partition of the foam volume. This uniqueness is a major advantage compared to other tessellation schemes. Next, the cells and the throats are identified, using a new expanding non-concave bubble algorithm. The algorithm identifies the faces of tetrahedra that surround every cell, it identifies the throats, it identifies the tetrahedra that surround every throat, and it identifies the connectivity of the cellular structure.

To illustrate the method, we characterized and analyzed two generic random structures: a mono-disperse foam and a 3D Voronoi tessellation. We have demonstrated that our analysis can distinguish easily between the two structures, both on the topological and the geometric properties. The distributions of all the topological characteristics were broader in the Voronoi structure. The differences were especially clear on analyzing the distributions of two geometric character-istics: the quadron volumes and throat areas. In both cases the distributions of the two structures were markedly different.

One of the advantages of the characterization method is that it makes possible to apply the entropic analysis, formulated originally by Edwards and collaborators for granular systems,[22] and extended later to cellular struc-tures,[20,21,29] to disordered foam structures. In particular, the method quantifies explicitly a volume function, which is at the heart of the formalism, whose evaluation had been only approximate. The formulation of the volume function relies on the identification of basic volume elements – the quadrons – that can be constructed to tessellate the volume of the foam.

Our algorithm identifies all the quadrons and characterizes their properties, one of which is their volume. For the mono-disperse foam we have found that a γ function describes very well the distribution of quadron volumes. Using this form, we illustrated calculations of structural properties as expectation values over the partition function. In particular, we calculated the expectation values of the moments of the volume distribution per vertex. Other possible structural characteristics that can be computed by this approach would be the cross-section of the skeletal throats and the entire solid–void surface in pseudo-granular systems.

Several methods have been used recently to analyze foam structures using 3D tomography. These include, extraction of the pore network and solid network by image analysis such as the watershed algorithm[17] and statistical methods aiming at getting scalar descriptors that give simple statistical mea-sures,[19] e.g., integral-geometry morphological image analy-sis and Minkowski functionals. While, this paper presents a system of extracting pore network as well, it is different from the former type of methods due to the fact that the algorithm is topological in nature. The algorithm searches for a unique cellular structure that is defined by its topology. Indeed, part of the identification can be done using the methods presented in Vicente et al.,[17] however, the resulting network must be adapted to work with the entropic formalism. The final construction is not a skeleton (nodes connected by one dimensional links) but a system of tetrahedra that touch at the corners that represent the solid and between them a set of polyhedra (cells) that share faces which are skewed polygons (the throats). The difference in structure is also responsible to some differences of morphological properties such as the throat area and number of faces per cell. For example, the number of faces per cell as calculated in Kraynik et al.[30] corresponds to the number of throats in our case and not to the number of faces that include the triangular faces. Also, the number of edges per throat for mono-dispersed foam is similar to the number of edges per face.[30] In addition to the above results, the structure is used to obtain a partition of the volume of the foam into basic units that represent the degrees of freedom of the foam – the quadrons. These are used to construct a statistical framework that is based on partition functions.[20,21] This approach is quite new in foam structures and the statistics of the basic elements (quadrons) have not been known prior to our work. As for the latter, the Minkowski functionals give several statistical descriptors of porous structures that have simple intuitive interpretation. These are complementary to the two point correlation function and are easy to obtain and apply easily to any porous structure. As such they are suitable for fast preliminary characterization of the structure. Nevertheless the statistics the Minkowski functional present is limited and the functional do not present tools to predict other properties of the complex structure. The entropic formalism presented here enables us to calculate properties of the foam as expectation values over the partition function. In addition, the polyhedral structure we obtain can be used directly to obtain properties of the porous

system, such as permeability or heat conduction. Nevertheless, our methods are harder to implement and thus less suitable for a quick survey of very large systems. Thus the two approaches should be looked upon as complementary to each other.

To conclude, we have demonstrated the applicability of an automated set of algorithms for the characterization and analysis of structures of foams and tetrahedral pseudo-granular systems. The characterization is compatible with the Edwards entropic formalism and to demonstrate its use we considered a simplified uncorrelated system and calculated the expectation values of moments of the quadron volume using the canonical ensemble. This is the first stage in the characterization. Taking into consideration the shape of the plateau borders and vertices as additional degrees of freedom is the next stage.

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